

# Advances in Material Modeling of Concrete

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## ABSTRACT

The paper gives a survey of the principal advances in material research in concrete presented at nine international conferences on Structural Mechanics in Reactor Technology since their inception in 1971. The survey focuses on nonlinear triaxial constitutive models, cracking and strain-softening damage, fracture, creep, hydrothermal deformations, heat transfer and moisture transfer. Although the survey is far from exhaustive, several fundamental advances are identified and discussed in some detail, although in nonmathematical terms. One hundred references are included.

## 1. INTRODUCTION

The tenth International Conference on Structural Mechanics in Reactor Technology (SMiRT) in Los Angeles is a milestone at which it is fitting to look back and appraise the evolution that has taken place in our understanding of the mechanical behavior of materials since the first conference in Berlin in 1971. The accomplishments, however, are too numerous, the subject too vast. Therefore, the present lecture, which is sponsored by Divisions Q and L of the Society, will focus on the mechanical behavior of concrete, and the broad and important field of metals will be relegated to a separate survey planned for the subsequent conference. The exposition which follows will include some general reflections on the historical evolution, as well as more detailed discussion of some selected highlights and interesting points dealing with nonlinear triaxial behavior, failure criteria, cracking, fracture, damage, creep, shrinkage, thermal strains, moisture and thermal effects, and diffusion. The references and bibliography will be limited to the contributions presented at SMiRT conferences as well as the closely connected journal Nuclear Engineering and Design in which many SMiRT conference contributions were published in greater detail. No claim is made for exhaustiveness of this literature survey; in fact, those authors who are not quoted will still be in good company.

## 2. SOME REFLECTIONS ON THE EVOLUTION SINCE THE FIRST CONFERENCE

Many of us still vividly remember the feeling of excitement at the first conference in Berlin, 1971. Thomas Jaeger gathered there the most impressive assembly of experts, and it seemed that if only their knowledge of material behavior were translated into finite element programs which had just become available, all the mechanics problems of design could be readily answered. However, this did not turn out to be the case, mainly because of the complexities of material modeling. The pitfalls of material softening after passing the peak stress (or the so-called failure envelope) were

as if they acted on the entire cross section of the structure instantaneously, neglecting diffusion, while for the purpose of shrinkage calculation the migration of moisture was described by linear diffusion theory. Effects of very high temperatures on concrete were ignored. The only size effect in structural response was thought to be statistical. We have indeed come a long way from these simplistic concepts, as the following discussion will try to document.

The advances of the research community in concrete were without delay reflected at the SMiRT conferences, as well as stimulated by them, although the important role of other meetings and especially workshops must not be overlooked. Throughout the 1970's, the progress was primarily driven by the needs of the expanding nuclear power programs, which justified lavish funding. Later, the thrust of research shifted from design for service conditions to safety evaluations. This forced serious attention to be paid to fracture mechanics aspects and post-peak energy dissipation, as well as the effects of very high temperature, exposure to liquid sodium or molten steel, etc. In recent years, despite the generally acknowledged necessity of nuclear power, and because of political pressures and choices made possible by very successful energy conservation, we have seen a retrenchment in public funding in nuclear reactor research. Fortunately, however, the community of researchers formerly supported by nuclear reactor programs continued to receive funds for basic and applied research in concrete which is useful to many technologies and objectives, nuclear power included. Thus, even at present we witness a rapid evolution in material modeling of concrete, which will make it possible to design better and safer structures for nuclear reactors when the need for further expansion becomes inevitable.

### 3. NONLINEAR TRIAXIAL BEHAVIOR AND FAILURE CRITERIA

The failure surface of concrete was relatively well known already at SMiRT1; e.g. Janda (1); formulas have been given for the rounded triangular shape of the deviatoric cross section, causing the tensile and compression meridians in the volumetric (Rendulić) cross section to be non-coincident (Fig. 1), as evidenced by the test results of Launay and Gachon (2). The biaxial failure envelopes at various values of the third stress were also known. However, it was not really known how to relate these failure surfaces to stress-strain relations and how to take into account the path dependence of concrete. Considerable complexities were later revealed by various new types of tests, e.g. on cubical triaxial specimens; Aschl and Moosecker (3); Valente (4); Robutti et al. (5); Ohnuma and Aoyagi (7); Watabe et al. (8), etc.

The first nonlinear incremental triaxial stress-strain relations were based on classical plasticity, especially the Drucker-Prager yield criterion, coupled with normality. However soon a variety of other approaches appeared. Description of a broad variety of test data was achieved in 1975 by Bažant (9) and coworkers, who developed for concrete the endochronic theory, extending ideas advanced previously for metals by Schapery and especially Valanis (Fig. 2). Other nonlinear triaxial models later appeared and achieved approximately the same closeness of fit of the existing data. These included the plastic-fracturing theory, which extended Dougill's previous idea for strain-space plasticity total strain models, and incremental orthotropic models. The latter ones however fell out of favor in recent years after a criticism of their problems with the form invariance restrictions. The incremental plasticity continued to be improved by refinements of the loading surfaces and the flow rule. An excellent description of a variety of test results has recently been achieved in the 1987 plasticity model by Chern et al., which introduced the volumetric cross section in the form of an expanding and shrinking slanted ellipse. In view of earthquake effects, and also in view of the shift of attention to the analysis of nuclear accidents, models for cyclic hysteretic stress-strain relations were formulated and rate effects introduced; e.g. Chen and Fardis (10); Soon, Buyukozturk and Einstein (11); Wang and Subia (12); Suaris and

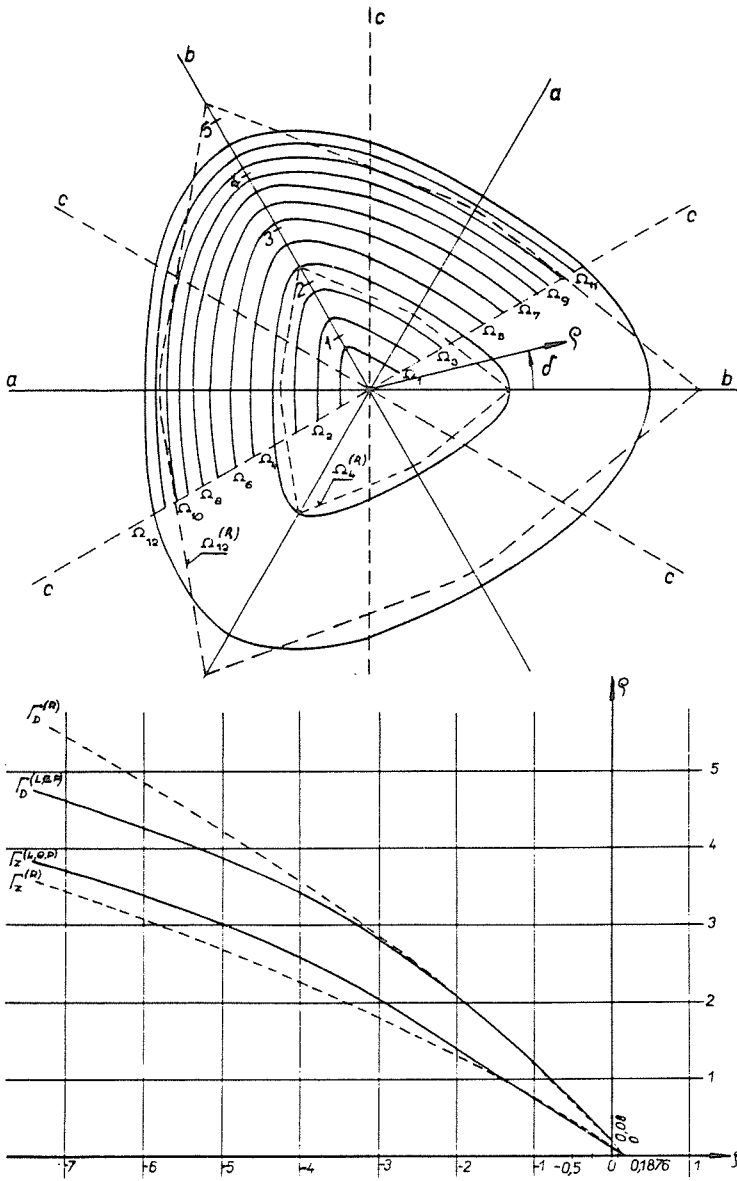


Fig. 1 Deviatoric (top) and volumetric (bottom) sections of the failure surface of concrete as presented by Janda (1).

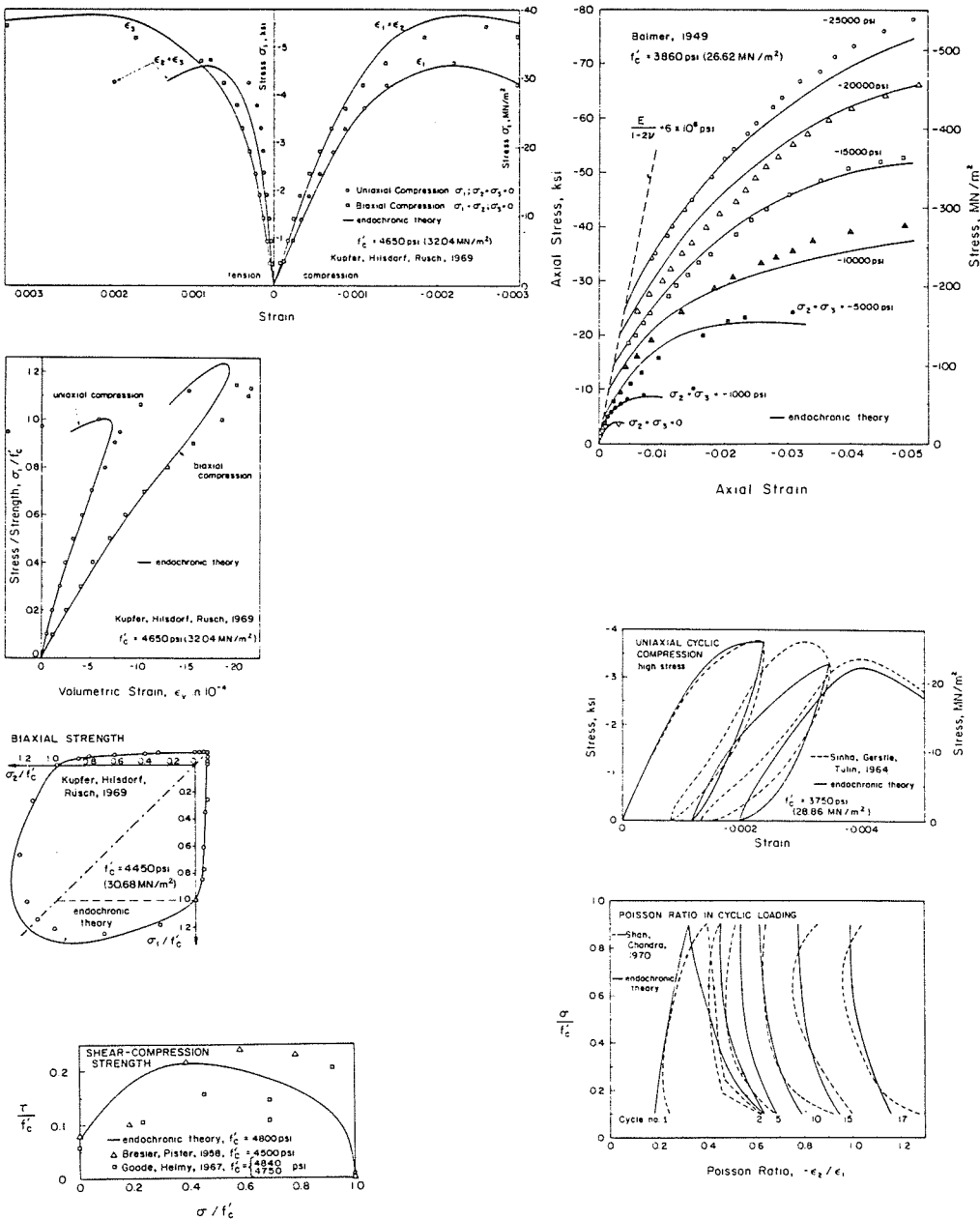


Fig. 2 Example of an early representation of a broad range of multiaxial stress-strain curves and failure envelopes by endochronic theory, presented in 1975 at SMIRT3 (9).

Shah (13); Eibl and Curbach (14); and Zeller (15). Numerical advantages and difficulties of various constitutive models were intensely discussed at SMiRT conferences. Problems related to stability, convergence and continuity of response were detected and partially handled first for the endochronic theory and later for plasticity without normality as well as other models.

After two decades of research, it seems that further improvements in the aforementioned nonlinear triaxial models, which may be needed especially if new test results appear for various nonproportional loadings, will be very difficult to achieve. This is probably true not only of classical plasticity but also of the plastic-fracturing models and the endochronic theory, as well as the continuum damage mechanics models, which appeared on the scene recently. The reason is that all these models are tensorial and essentially phenomenologic, ignoring the phenomena which cause inelastic behavior in the microstructure. Therefore, the promise of further progress probably lies in micromechanics modeling, which is simpler (nontensorial) and can take into account aggregate (particle) interactions and cracking, separating the phenomena occurring in the microstructure on planes of various orientations.

Although some aspects of micromechanics were discussed since the first conference; e.g. Wittmann and Zaitsev (16), the subject did not take off until recently. Aided by a powerful computer, Roelfstra (17), and Wittmann and Roelfstra (18), achieved a much better understanding of the micromechanism of inelastic behavior by their finite element analysis of stresses and deformations in the microstructure. Considerable potential is no doubt offered by rigid particle models of the type introduced first by Cundall for sands, which were recently generalized for concrete; e.g. by Zubelewicz and Bazant.

As a modeling tool for finite element programs, the microplane approach seems to have a particular potential. This approach is based on a 1938 idea of Taylor according to which the material behavior may be specified by stress-strain relations for the components of stresses and strains acting on planes of various orientations in the microstructure, called the microplanes. These are then combined according to a weak variational principle into a macroscopic stress-strain relation. By virtue of the fact that the stress-strain relation on the microplane does not have to deal with tensor components and invariance, the number of variables is greatly reduced and conceptual simplicity achieved (the tensorial invariance requirements are automatically satisfied by combining the responses from the planes of all orientations). A microplane model of this type was first developed for tensile cracking in 1973 by Bazant and Oh and later extended by Gambarova and Floris (19). It was also shown capable of modeling the resistance of cracks to shear. Future development should probably also take into account recent successes in the modeling of microcrack arrays, although for concrete this would be compounded by the difficulties of the aggregate structure.

#### 4. CRACKING

In the early studies, concrete was assumed to have no tensile resistance. This idealization conveniently circumvented all the difficulties associated with the stress drop after cracking. However, in many types of failure, generally all the so-called brittle failures of concrete structures, the concept of a no-tension material grossly underestimates the carrying capacity, and even more the energy absorption capacity. A pioneering idea occurred to Rashid (20) in his famous paper in Nuclear Engineering and Design, in which he proposed to consider cracking to be smeared and handle it by a reduction of material stiffness in the cracked finite element, coupled with a stress drop to zero. This idea was further extended in 1971 in Scanlon's dissertation, who recognized the need to provide for a gradual

stress decrease, i.e. strain softening, and modeled it by successive drops in stiffness. Aside from the modeling of cracks as interelement cracks, introduced in 1967 by Ngo and Scordelis, the concept of smeared cracking became particularly popular in the analysis of nuclear reactor vessels and containments and led to many analytical successes.

These early successes can for example be documented by the analysis of the spread of cracking in the cross section of a prestressed concrete pressure vessel reported by Zienkiewicz, Owen and Nayak (21), see Fig. 3, or the dynamic analysis of the cracking sequence in a prestressed concrete reactor vessel for a sodium-cooled breeder reactor, obtained in a hypothetical core disruptive accident; Marchertas et al. (22), see Fig. 4. As another of the early successes of complex nonlinear finite element analysis, one should mention the work of Rebora, Zimmermann and Wolf (23), in which a hypothetical aircraft crash onto a containment shell was analyzed using a sophisticated plasticity model with noncircular deviatoric cross sections; see Fig. 5 (cf. also Zimmermann (100)).

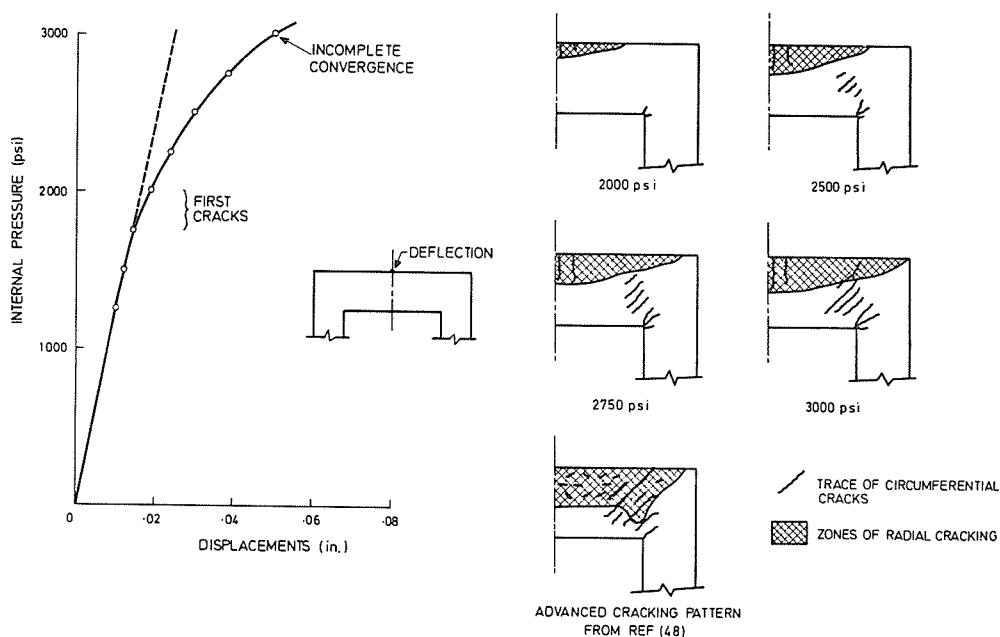


Fig. 3 Early finite results of Zienkiewicz et al. (21) showing the load-deflection diagram and the spread of cracking zones in a prestressed concrete pressure vessel.

The smeared cracking approach, however, was soon discovered to have some serious problems which may (although need not) make the solution invalid. As transpired already in the discussions between Willam and Bažant at SMiRT 3, London, the diagrams of load versus length of the cracking zone or load versus load-point displacements, and even more importantly the amount of dissipated energy, do not exhibit correct convergence as the mesh is refined. The results suffer from spurious mesh sensitivity. This is documented by Fig. 6 which shows the results of a rectangular panel in which a smeared cracking band propagates from an initially weaker element, the front of the cracking band being always localized to a single finite element. Subsequent years saw lively debates, first about the significance

of the problem, and after this was generally recognized, about the remedies. Many kinds of remedies were proposed and examined. The successful ones, however, all contained the basic features of fracture mechanics.

### 5. FRACTURE, DAMAGE, AND STRAIN-SOFTENING

Fracture mechanics is a failure theory which (1) determines failure on the basis of energy criteria or combined energy and strength criteria, and (2) takes into account the fact that the failure is not simultaneous in various parts of the structure but propagates through the structure. From the practical viewpoint, the most important consequence of the fracture mechanics aspects of failure is the size effect, which is related to the spurious mesh sensitivity in finite element calculations already mentioned.

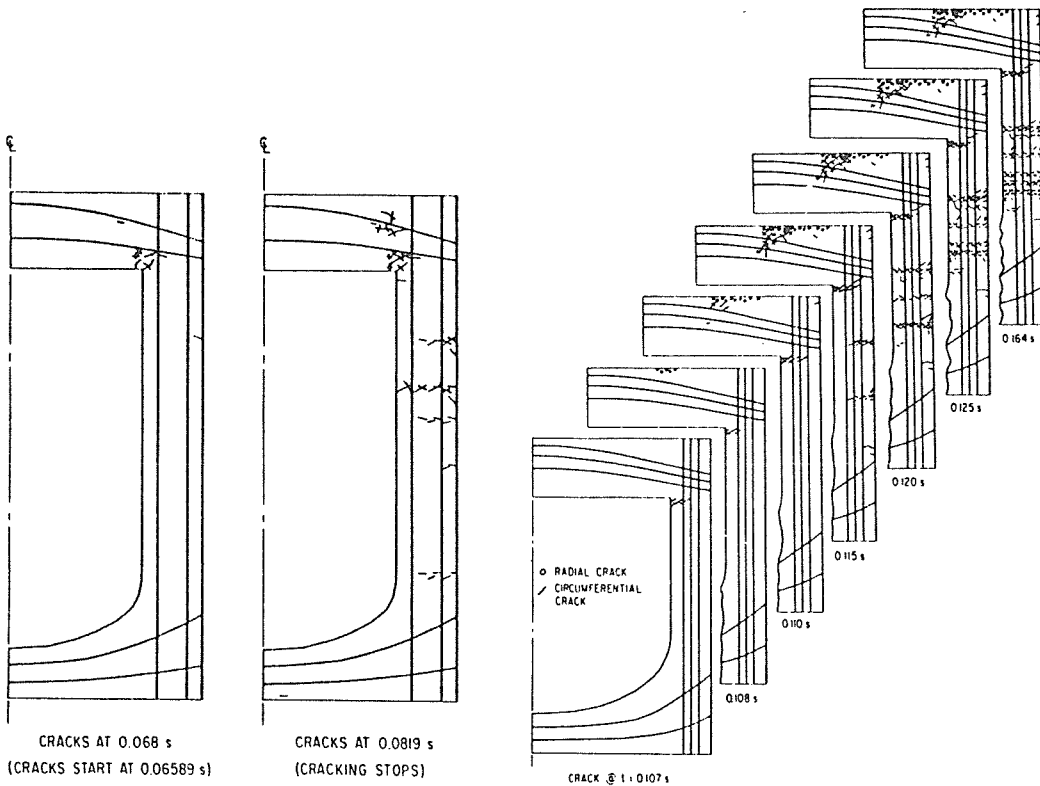


Fig. 4 Finite element results of Marchertas et al. (26) for the cracking pattern caused by a hypothetical core-disruptive accident in a prestressed concrete guard vessel for a sodium-cooled reactor.

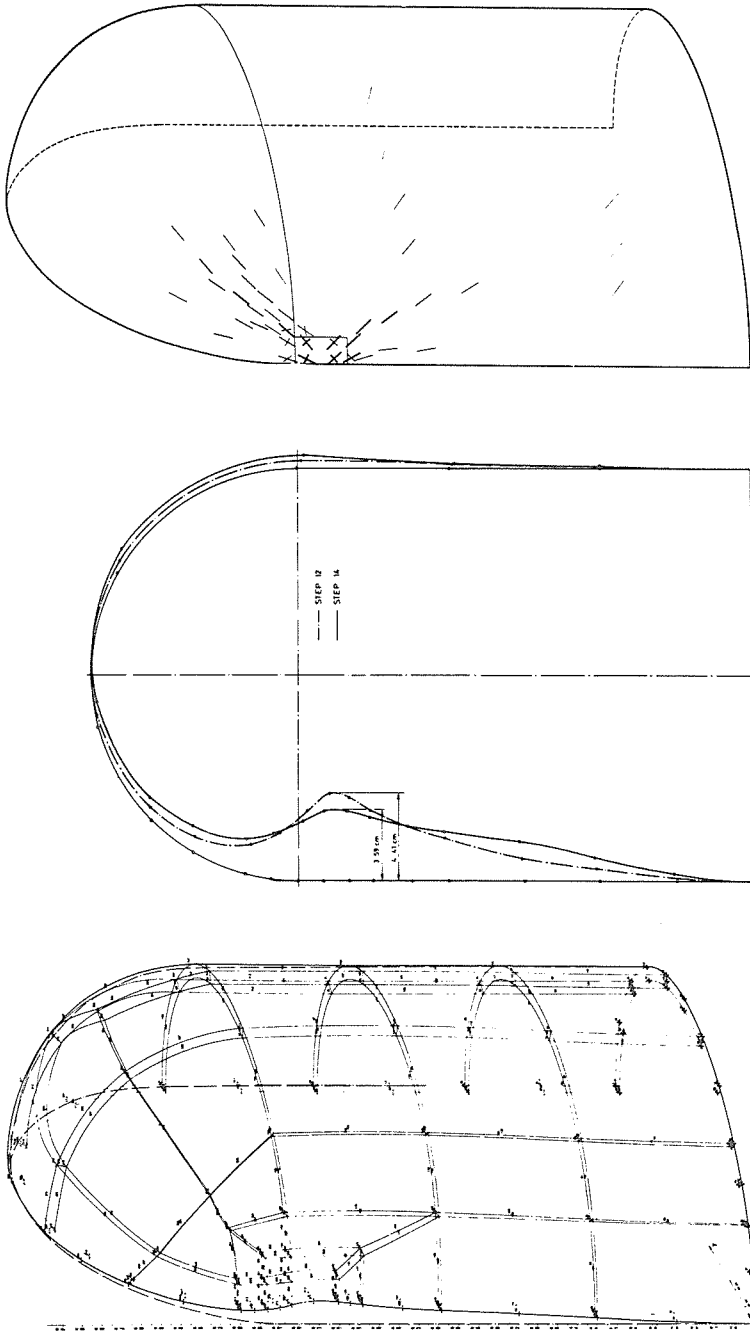


Fig. 5 Finite element results of Rebora, Zimmermann and Wolf (23) for impact of an aircraft into a containment shell; mesh (left), deflection profiles (middle) and cracking pattern (right).

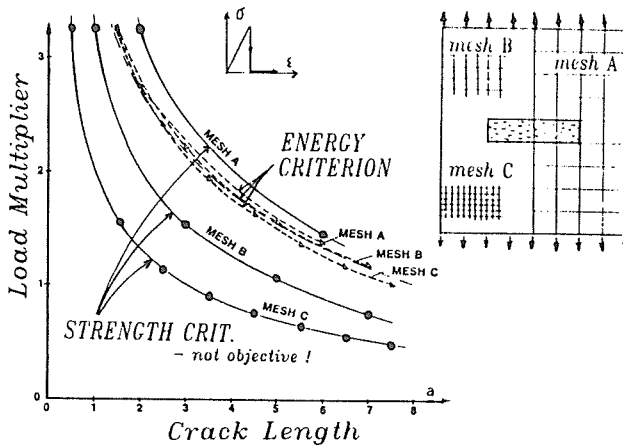


Fig. 6 Example of propagation of cracking band in a rectangular panel under tension, analysed by meshes A, B, C of sizes 4:2:1 (24).

The size effect is defined by comparing geometrically similar structures of different sizes. According to any theory with a strength criterion or a failure surface in the stress or strain space, such as the plastic limit analysis or elastic allowable stress design, geometrically similar structures of different sizes fail at the same value of the nominal stress, i.e. the failure shows no size effect. Not so in structures which exhibit fracture, either localized or distributed, the latter case being called the strain-softening.

The size effect began to be debated intensely at SMiRT conferences in the early 1980's, and a thorough discussion was presented at SMiRT 7 by Bazant (24); see Fig. 7, which shows a typical plot of the nominal stress at failure,  $N$ , versus the relative structure size,  $d$ , normalized with respect to the aggregate size,  $d_a$ , in logarithmic scales. As this plot reveals, the size effect is small for small structure sizes, and strength criteria for plastic limit analysis are then adequate. By contrast, for very large structure sizes, the size effect plot asymptotically approaches a straight line of slope -0.5, which is exactly the size effect of linear elastic fracture mechanics, a theory in which all the fracture process is assumed to be concentrated in a point and the rest of the structure to behave elastically. This size effect is the strongest possible. Real structures exhibit a transitional size effect between plastic limit analysis and linear elastic fracture mechanics.

The size effect can be exploited for determining the basic nonlinear fracture parameters, including the fracture energy  $G_f$  (32), the elastically equivalent size of the fracture process zone  $c_f$ , and the crack tip opening displacement, as well as the R-curve (Fig. 7 top right). Size effect measurements make it possible to determine the dependence of fracture energy on temperature and on moisture content.