

Nonlinear Creep Analysis of Prestressed Concrete Structures

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1. INTRODUCTION

Nonlinear behaviour of reinforced concrete structures has been the object of intensive theoretical and experimental investigations. Recent results of research in this field were presented at many conferences, e.g. IABSE Delft 1981, Split 1984, Karpacz 1984, Bombay 1985, Tokyo 1986, Tucson 1987, SMiRT Lausanne 1987. Some problems connected with cracking, elastic-plastic behaviour of reinforcement and concrete at the long-term loading and creep were considered by Bazant et al, (1979), Karpenko and Pietrov (1976, 1980), Szarliński et al, (1986) and others.

The aim of the paper is to present a model of prestressed concrete structures in the uniaxial and two-axial stress states in which the elastic properties, linear and nonlinear creep strains and shrinkage of concrete are accounted for together with the nonlinear strains of reinforcement and cracking. As an example, calculations of prestressed concrete beam are given and compared with the experimental evidence.

2. UNIAXIAL STRESS STATE

The equilibrium equation for an uniaxially stressed reinforced concrete prism can be written as

$$(1) \quad \sigma - \mu \sigma_0 = E'_b (\varepsilon - \varepsilon_0) + E_s \mu \varepsilon, \quad \text{where}$$

σ stress in the prism from the external load,
 σ_0 initial stress in reinforcement due to prestressing,
 ε strain in the prism,
 ε_0 initial strain in concrete,
 E'_b secant modulus of concrete,
 E_s Young's modulus of reinforcement,
 μ^s steel ratio.

Let us consider the first increment of applied stress σ_1 starting at time τ_1 and the shrinkage strain curve for concrete ε_s as shown in Fig. 1. Strain in the prism and stress in the concrete at time τ_1 can be obtained from the following formulae:

$$(2) \quad \varepsilon = (\sigma_1 - \sigma_0 \mu + E'_b \varepsilon_0) / (E'_b + E_s \mu),$$

$$(3) \quad \sigma_b = E'_b (\varepsilon - \varepsilon_0),$$

where $E'_b = E_b(\tau_1)$ is the elastic modulus of concrete at the age τ_1 and $\varepsilon_0 = \varepsilon_s(\tau_1)$.

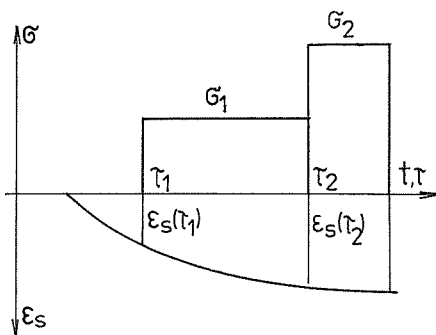


Fig. 1. Loading steps and shrinkage strain curve

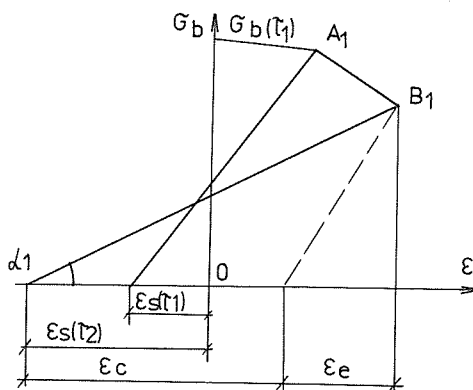


Fig. 2. Change of stress-strain state due to creep and shrinkage of concrete

The degradation of the stress-strain state of the prism caused by creep and shrinkage of concrete is shown in the diagram of Fig. 2. Prediction of the stress-strain state at time τ_2 requires the following iterative process to be used:

$$\begin{aligned}
 \epsilon_e + \epsilon_c &= \sigma_b / E_b(\tau_2) + \epsilon_c(\tau_2), \\
 E'_b &= \sigma_b / (\epsilon_e + \epsilon_c), \\
 \epsilon &= (\sigma_1 - \sigma_0 \mu + E'_b \epsilon_0) / (E'_b + E_s \mu), \\
 \sigma_b &= E'_b (\epsilon - \epsilon_0), \quad \text{where } \epsilon_0 = \epsilon_s(\tau_2).
 \end{aligned}
 \tag{4}$$

The formula (4)₁ gives the global strain in concrete which consists of linear elastic strain ϵ_e and creep strain ϵ_c . The expression (4)₂ allows to calculate the secant modulus of concrete at time τ_2 which equals $\text{tg} \alpha_1$, Fig. 2. The formula (4)₃ gives the strain in the prism and (4)₄ supplies stress in the concrete at time τ_2 . It is necessary to repeat the cycle (4)₁-(4)₄ as long as the strain in the prism ϵ or stress in concrete σ_b ceases to change appreciably.

At an instant of the change of stress σ the increment of stress and strain in concrete will have an elastic character and can be obtained from the following formulae:

$$\Delta \epsilon = (\sigma_2 - \sigma_1) / [E_b(\tau_2) + E_s \mu],
 \tag{5}$$

$$\sigma_b = \sigma_b(\tau_2) + \Delta \epsilon E_b(\tau_2).
 \tag{6}$$

For the analysis of the stress-strain state at time t it is possible to follow cycle (4) provided time τ_2 is replaced by time t .

In the case of change of sign of stress in concrete, creep strains at compression and tension can not be added. One part is used for the calculation of secant modulus E'_b and another is added to the shrinkage strain in the formula for ϵ_0 .

If tensile stress in concrete attains its ultimate strength the cracks begin to form. The continuous model is used to describe reinforced concrete with cracks. On this assumption the equilibrium equation of the prism can be written as