

Nonlinear Creep Analysis of Prestressed Concrete Structures

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1. INTRODUCTION

Nonlinear behaviour of reinforced concrete structures has been the object of intensive theoretical and experimental investigations. Recent results of research in this field were presented at many conferences, e.g. IABSE Delft 1981, Split 1984, Karpacz 1984, Bombay 1985, Tokyo 1986, Tucson 1987, SMiRT Lausanne 1987. Some problems connected with cracking, elastic-plastic behaviour of reinforcement and concrete at the long-term loading and creep were considered by Bazant et al, (1979), Karpenko and Pietrov (1976, 1980), Szarliński et al, (1986) and others.

The aim of the paper is to present a model of prestressed concrete structures in the uniaxial and two-axial stress states in which the elastic properties, linear and nonlinear creep strains and shrinkage of concrete are accounted for together with the nonlinear strains of reinforcement and cracking. As an example, calculations of prestressed concrete beam are given and compared with the experimental evidence.

2. UNIAXIAL STRESS STATE

The equilibrium equation for an uniaxially stressed reinforced concrete prism can be written as

$$(1) \quad \sigma - \mu \sigma_0 = E'_b (\varepsilon - \varepsilon_0) + E_s \mu \varepsilon, \quad \text{where}$$

σ stress in the prism from the external load,
 σ_0 initial stress in reinforcement due to prestressing,
 ε strain in the prism,
 ε_0 initial strain in concrete,
 E'_b secant modulus of concrete,
 E_s Young's modulus of reinforcement,
 μ^s steel ratio.

Let us consider the first increment of applied stress σ_1 starting at time τ_1 and the shrinkage strain curve for concrete ε_s as shown in Fig. 1. Strain in the prism and stress in the concrete at time τ_1 can be obtained from the following formulae:

$$(2) \quad \varepsilon = (\sigma_1 - \sigma_0 \mu + E'_b \varepsilon_0) / (E'_b + E_s \mu),$$

$$(3) \quad \sigma_b = E'_b (\varepsilon - \varepsilon_0),$$

where $E'_b = E_b(\tau_1)$ is the elastic modulus of concrete at the age τ_1 and $\varepsilon_0 = \varepsilon_s(\tau_1)$.

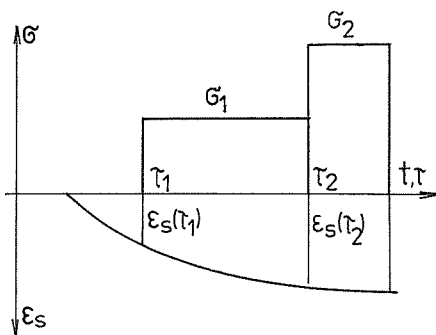


Fig. 1. Loading steps and shrinkage strain curve

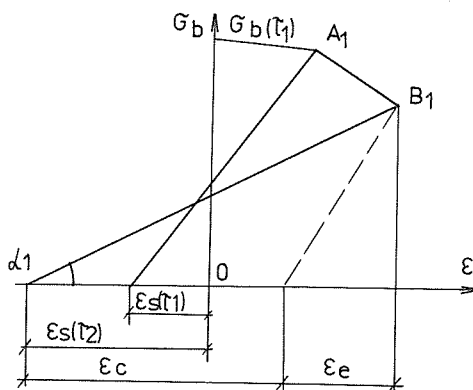


Fig. 2. Change of stress-strain state due to creep and shrinkage of concrete

The degradation of the stress-strain state of the prism caused by creep and shrinkage of concrete is shown in the diagram of Fig. 2. Prediction of the stress-strain state at time τ_2 requires the following iterative process to be used:

$$\begin{aligned}
 \epsilon_e + \epsilon_c &= \sigma_b / E_b(\tau_2) + \epsilon_c(\tau_2), \\
 E'_b &= \sigma_b / (\epsilon_e + \epsilon_c), \\
 \epsilon &= (\sigma_1 - \sigma_0 \mu + E'_b \epsilon_0) / (E'_b + E_s \mu), \\
 \sigma_b &= E'_b (\epsilon - \epsilon_0), \quad \text{where } \epsilon_0 = \epsilon_s(\tau_2).
 \end{aligned}
 \tag{4}$$

The formula (4)₁ gives the global strain in concrete which consists of linear elastic strain ϵ_e and creep strain ϵ_c . The expression (4)₂ allows to calculate the secant modulus of concrete at time τ_2 which equals $\text{tg} \alpha_1$, Fig. 2. The formula (4)₃ gives the strain in the prism and (4)₄ supplies stress in the concrete at time τ_2 . It is necessary to repeat the cycle (4)₁-(4)₄ as long as the strain in the prism ϵ or stress in concrete σ_b ceases to change appreciably.

At an instant of the change of stress σ the increment of stress and strain in concrete will have an elastic character and can be obtained from the following formulae:

$$\Delta \epsilon = (\sigma_2 - \sigma_1) / [E_b(\tau_2) + E_s \mu],
 \tag{5}$$

$$\sigma_b = \sigma_b(\tau_2) + \Delta \epsilon E_b(\tau_2).
 \tag{6}$$

For the analysis of the stress-strain state at time t it is possible to follow cycle (4) provided time τ_2 is replaced by time t .

In the case of change of sign of stress in concrete, creep strains at compression and tension can not be added. One part is used for the calculation of secant modulus E'_b and another is added to the shrinkage strain in the formula for ϵ_0 .

If tensile stress in concrete attains its ultimate strength the cracks begin to form. The continuous model is used to describe reinforced concrete with cracks. On this assumption the equilibrium equation of the prism can be written as

$$(15) \quad \varepsilon_n(t) = \int_0^{\sigma_{max}} \frac{\partial \phi(\sigma, \tau+T, \tau)}{\partial \tau} d\sigma,$$

where T is duration of the action of $d\sigma$. The function $\phi(\sigma, t, \tau)$ is the product of stress in concrete and nonlinear creep strain per unit stress (creep function), i.e.

$$(16) \quad \phi(\sigma, t, \tau) = \sigma^r C_n(t, \tau), \quad r > 1.$$

Some forms of specific creep function for concrete ensuring a fairly good agreement with experimental data are given by Aleksandrowskij (1966).

An attempt to employ the model of concrete based on the integral forms (14) and (15) leads to very cumbersome computer calculations. The specific τ_t - mode of creep prediction, proposed by Karpenko and Pietrov (1979, 1980), allows to simplify the calculations.

3. TWO-AXIAL STRESS STATE

Plane concrete element reinforced in the x, y - directions is considered. Reinforcing bars in both directions are smeared out and characterized by steel ratios μ_x and μ_y . Prestressing bars with steel ratio μ_n are embedded in the x direction only. Constitutive equations for the x, y directions have the form

$$(17) \quad \{\sigma\} = [D]_{bs} \{\varepsilon\} - [D]_b \{\varepsilon_0\},$$

where $[D]_{bs}$ and $[D]_b$ are stiffness matrices of reinforced and plain concrete, respectively. Initial strains in concrete are calculated from the formulae

$$(18) \quad \{\varepsilon_0\} = \begin{Bmatrix} \varepsilon_s + \varepsilon_{cn} \sin^2 \gamma + \varepsilon_{ct} \cos^2 \gamma \\ \varepsilon_s + \varepsilon_{cn} \cos^2 \gamma + \varepsilon_{ct} \sin^2 \gamma \\ (\varepsilon_{cn} - \varepsilon_{ct}) \sin \gamma \cos \gamma \end{Bmatrix},$$

where ε_s denotes shrinkage strain and ε_{cn} , ε_{ct} are creep strains in concrete in the principal directions n, t . Directions of principal stresses n, t in concrete are shown in Fig. 4. The orthotropic model is used for strained concrete before cracking. Elements of matrix $[C]_b = [D]_b^{-1}$ are calculated from the relations.

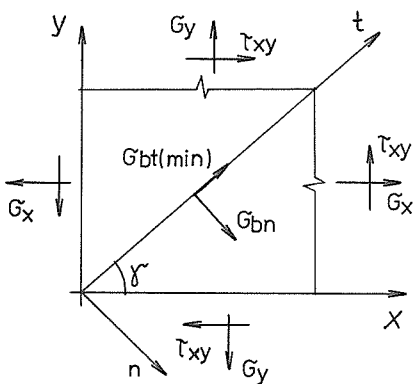


Fig. 4. Directions of principal stresses n, t in concrete

$$(19) \quad \begin{aligned} C_{11b} &= \sin^2 \gamma / E'_{bn} + \cos^2 \gamma / E'_{bt}, \\ C_{12b} &= -\nu_b (1+m) / (mE'_{bn} + E'_{bt}), \\ C_{13b} &= C_{23b} = (1/E'_{bn} - 1/E'_{bt}) \sin \gamma \cos \gamma, \\ C_{22b} &= \cos^2 \gamma / E'_{bn} + \sin^2 \gamma / E'_{bt}, \\ C_{33b} &= 1/E'_{bn} + 1/E'_{bt} + 2\nu_b (1+m) / (mE'_{bn} + E'_{bt}), \end{aligned}$$

where

$$\gamma = 0,5 \operatorname{arc} \operatorname{tg} \left[\frac{2\tau_{bxy}}{(\sigma_{bx} - \sigma_{by})} \right],$$

$$m = \left| \frac{\sigma_{bn}}{\sigma_{bt}} \right| \leq 1 \quad \text{and} \quad \nu_b$$

denotes Poisson's ratio (here assumed 0,2). In the stiffness matrix for reinforced concrete only two elements differ from those for the plain concrete:

$$(20) \quad D_{11bs} = D_{11b} + \mu_x E_{sx} + \mu_n E_{sn} ,$$

$$D_{22bs} = D_{22b} + \mu_y E_{sy} .$$

The secant moduli of concrete E'_{bn} , E'_{bt} , present in (19), are calculated in accordance with the iterative process (4) given in Section 2 of the paper. Both the iterative process (4) and the initial strains formulae (18) contain creep strains of concrete in principal directions. To take into account the influence of two-axial stress upon creep the following formula is proposed

$$(21) \quad \epsilon_{c\alpha} = \delta_{b\alpha} \left[C_l^*(t, \tau) + k_\alpha \delta_{b\alpha}^r C_n(t, \tau) \right], \quad \alpha = n, t ,$$

where $C_l^*(t, \tau)$ is a specific linear creep which is to account for the two-dimensionality of the element, k_α denotes empirical coefficient accounting for the effect of two-axial stress on the nonlinear creep. For determining k_α the following formulae are postulated:

at the "compression-compression" state

$$k_n = k_t = 1 - 0,7 \sqrt{|\delta_{bn}| / R_b(\tau)} ,$$

where $R_b(\tau)$ stands for cube strength of concrete at the age τ , at the "compression-tension" state

$$k_n = \left[1 + 0,9 \delta_{bt} / R_b(\tau) \right]^{-1}, \quad (\delta_{bt} < 0), \quad k_t = 1,$$

at the "tension-tension" state

$$k_n = k_t = 1.$$

The anisotropic model is used for reinforced concrete with cracks. Matrix $[D]_{bs} = [C]_{bs}$ in (17) can be obtained from the following formulae based on Karpenko (1976) suggestions:

$$(22) \quad C_{11bs} = \lambda_x / (E'_{sx} \mu_x + E'_{sn} \mu_n) + \cos^2 \gamma / E'_{bt} ,$$

$$C_{12bs} = 0 ,$$

$$C_{13bs} = \lambda_x \operatorname{ctg} \gamma / (E'_{sx} \mu_x + E'_{sn} \mu_n) - \sin \gamma \cos \gamma / E'_{bt} ,$$

$$C_{22bs} = \lambda_y / E'_{sy} \mu_y + \sin^2 \gamma / E'_{bt} ,$$

$$C_{23bs} = \lambda_y \operatorname{tg} \gamma / E'_{sy} \mu_y - \sin \gamma \cos \gamma / E'_{bt} ,$$

$$C_{33bs} = \lambda_x \operatorname{ctg}^2 \gamma / (E'_{sx} \mu_x + E'_{sn} \mu_n) + \lambda_y \operatorname{tg}^2 \gamma / E'_{sy} \mu_y + (1/\nu_p - 1) / E'_{bt} ,$$

where

$$1/\lambda_x = 1 + E'_{sy} \mu_y \operatorname{ctg}^2 \gamma / (E'_{sx} \mu_x + E'_{sn} \mu_n) ,$$

$$1/\lambda_y = 1 + (E'_{sx} \mu_x + E'_{sn} \mu_n) \operatorname{tg}^2 \gamma / E'_{sy} \mu_y ,$$

$$E'_{s\alpha} = E_{s\alpha} / \psi_{s\alpha} / \psi_{\underline{s}\alpha} , \quad \alpha = x, y, n ,$$

$$\gamma_p = 0,15 + \left[0,025 R_b(\tau) / R_{b\text{ten}}(\tau) / \sigma_{bn} \right]^2 \leq 0,6.$$

Coefficients $\psi_{s\alpha}$, $\psi_{g\alpha}$ are determined in accordance with (9) and (13). The stiffness matrix of concrete in (17) is obtained from the formula

$$(23) \quad [D]_b = [D]_{bs} - [D]_s,$$

where $[D]_s = [C]_s^{-1}$; Matrix $[C]_s$ is calculated in accordance with (22) on the assumption that $E_{s\alpha} = E_{s\alpha} \gamma_{s\alpha}$, $\alpha = x, y, n$.

4. NUMERICAL EXAMPLE

As an example of the proposed method the FEM computer calculations of prestressed concrete beam, experimentally investigated in Moscow Research Concrete Institute, were performed.

All material functions for concrete used in the numerical analysis were determined by Chernozharova (1971) from the experiments in the following form:

$$(24) \quad \begin{aligned} \epsilon_s(\tau) &= 25 \cdot 10^{-5} (1 - e^{-0,4\tau}), \\ E_b(\tau) &= 3,6 \cdot 10^4 (1 - e^{-0,1\tau}), \quad (\text{MPa}), \\ R_b(\tau) &= 43 (1 - e^{-0,1\tau}), \quad (\text{MPa}), \\ R_{b\text{ten}}(\tau) &= 0,28 [R_b(\tau)]^{2/3}, \quad (\text{MPa}), \\ C_l(t, \tau) &= 8,4 \cdot 10^{-5} (0,1 + e^{-0,13\sqrt{t}}) (1 - e^{-0,13\sqrt{t-\tau}}), \quad (\text{MPa}^{-1}), \\ C_n^+(\sigma, t, \tau) &= [\sigma / R_{b\text{ten}}(\tau)] C_n(t, \tau), \quad (\text{MPa}^{-1}), \\ C_n^-(\sigma, t, \tau) &= [\sigma / R_b(\tau)] C_n(t, \tau), \quad (\text{MPa}^{-1}), \\ C_n(t, \tau) &= 3,8 \cdot 10^{-5} (1 - e^{-0,25\sqrt{t-\tau}}), \quad (\text{MPa}^{-1}), \end{aligned}$$

where t and τ denote time in days.

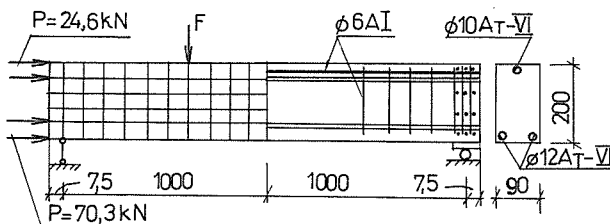


Fig. 5. Reinforcement of beam, finite element mesh and loading

Two symmetrical parts of the beam subdivided into rectangular finite elements are shown in Fig. 5. The initial flexure under prestressing, permanent deflection under the long-term loading under $F = 17,5$ kN and cracking process under the long-term loading $F = 17,5$ kN and cracking process under the short-term loading $F = 53,3$ kN are shown in fig. 6 and Fig. 7. A fairly good agreement with the experimental data can be observed.

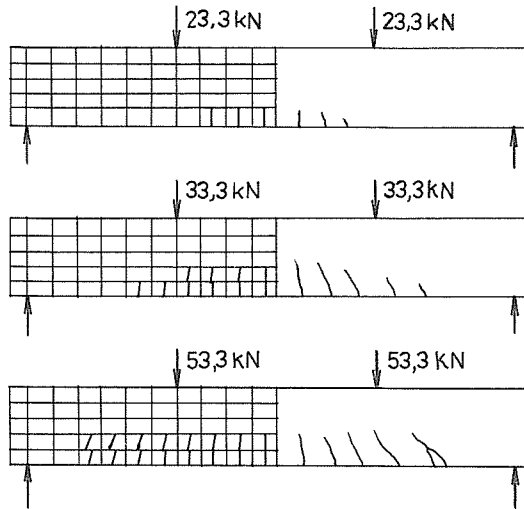
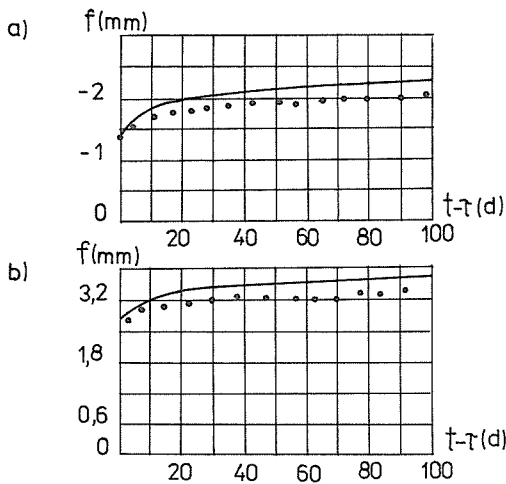


Fig. 6. a) Initial flexure of beam under prestressing
 b) Permanent deflection of beam under long-term-loading
 — theoretical results
 experimental results

Fig. 7. Cracking process in beam under short-time loading

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